Technical Comments

Comment on "Numerical Solution of the Boundary-Layer Equations"

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E. KRAUSE[1] stated that the Smith and Clutter[2] method for integrating the boundary-layer equations requires "not only u and T, but also $\partial u/\partial x$ and $\partial T/\partial x$ for the start of the integration." That this statement is incorrect can be demonstrated from the transformed momentum equation (we consider only incompressible flow for simplicity):

$$f''' + \left(\frac{M+1}{2}\right)ff'' - M(f'^2 - 1) = x\left[f'\frac{\partial f'}{\partial x} - f''\frac{\partial f}{\partial x}\right] \quad (1)$$

where $f' = df/d\eta = u/u_e$, $\eta = y(u_e/\nu x)^{1/2}$, and $M(x) = (x/u_e)(du_e/dx)$. Using a simple two-point finite-difference scheme for the x derivatives, Eq. (1) becomes

$$f''' + \left(\frac{M+1}{2}\right)ff'' - M(f'^2 - 1) = \left(\frac{x+\Delta x}{\Delta x}\right)[f'(f' - F') - f''(f - F)] \quad (2)$$

where $f = f(x + \Delta x, \eta)$ and $F = f(x,\eta)$.

Equation (2) can be solved for f, subject to the appropriate boundary conditions, once F and F' are known. If at a particular value of x $(=x_0)$, the initial velocity distribution u/u_e is known, then

$$F' = \frac{u}{u_e}(x_0,\eta) \qquad F = \int_0^\eta F'(x_0,\eta)d\eta \quad (3)$$

Equation (2) can then be used to determine the solution as one proceeds downstream using Eq. (3) as a starting condition. An analogous result applies to the compressible case.

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[1] Krause, E., "Numerical solution of the boundary-layer equations," AIAA J. 2, 1231–1237 (1967).
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IF only the tangential velocity component and temperature are prescribed at the initial station, a compatible normal velocity component can always be computed. This, for example, is demonstrated in Ref. 1 for the implicit finite-difference solution of Flügge-Lotz and Blottner.

The remark in Ref. 1 regarding Smith and Clutter's method is motivated by Eq. (9) of Ref. 2. That equation expresses the x-derivatives by means of a three-point difference formula, presumably to achieve high accuracy. When the three-point formula is used to start the integration, it is necessary to specify the initial data at two previous stations instead of one. This is equivalent to specifying the function and its x-derivative at one station. The suggestion of the comment to use a two-point difference formula for the x-derivative does, in principle, eliminate the difficulties in starting the integration.

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Comment on "A Source Model for Predicting the Drag Force on a Moving Arc Column"

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THE source model used by Otis[1] to describe an electric arc column in a crossflow, is open to criticism 1) on the validity of applying this particular model to an arc and 2) on comparison with experimental results. Concerning 1, the following two main points may be made:

a) The relation $2\pi m = U_\alpha b_w$, (Sec. IIC), applies only to fluid that has the same density on both sides of the dividing streamline shown in Fig. 3 of Ref. 1. It is incorrect to assume that the preceding relation holds when the fluid emitted by the source is heated, so that the densities on the two sides of the dividing streamline are unequal.

b) The incompressible inviscid point source model adopted in Sec. IIC, since it is both a mass source and a volume source, contradicts the assumption implicit in the derivation of the mass and energy equations in Sec. IIA, which is that all fluid originates upstream of the zone of heating.

Apart from these two criticisms, other assumptions that might be disputed are the neglect of heat conduction and radiation in the energy equation of Sec. II.

Comparison of the source model with experimental results (criticism 2) is most conveniently made by considering the following equation obtained from the model, which is the form in which Otis also makes such a comparison:

$$c_\alpha B/E \simeq (k - 1)M_\alpha \quad (1)$$

where c_α is the freestream speed of sound, B the transverse magnetic field, E the column voltage gradient, k the ratio of

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specific heats, and M_α the freestream Mach number such that $M_\alpha < 0.3$. Equation (1) relates to the freestream velocity $U_\alpha = c_\alpha M_\alpha$ in which an arc is held stationary by a transverse magnetic field, but it is generally assumed that U_α equals the velocity U of the arc in still gas, and it is known experimentally[2] that this is so at least up to spacings of about 3 cm and velocities up to 25 m/sec. It should be noted that Eq. (1) does not imply, as stated by Otis, that the relationship between flow velocity and magnetic field is independent of current I, since E depends on I. The experimental results with which it is compared, however, are for a very small current range, 30–75 amp, and the apparent independence of B on I is explained[3] by assuming that the effective arc width or diameter W is proportional to current I. This is not in accordance with recent results for an arc moving through argon at a pressure of 3 atm at spacings up to 1 cm, where over the range 50–700 amp it has been found[4] that $C_D W \propto I^a$ where $0.30 < a < 0.44$. This result suggests that Eq. (1) is incomplete, and this is confirmed by results for an arc held stationary in an air stream by a transverse magnetic field[5] as shown in Fig. 1, where different values of $c_\alpha B/E$ for a given arc velocity are obtained when the arc current is varied. Also it can be seen in Fig. 1 that the curves for these currents are not straight lines from the origin, nor do they fall near the line for $k = \frac{7}{5}$, the latter being predicted by the source model for diatomic gases. Further data[6] which do not agree with the source model are also presented in Fig. 1, for arcs driven through still air between rail electrodes 3.8 cm apart by a transverse field. The source model implies that all results should collapse onto a single line to the origin, and this does not occur. The results of Roman and Myers in Fig. 1 show that $c_\alpha B/E$ is higher for a given M_α, the lower the arc current. Experiments show that velocity $\propto B^m I^n$ and m is frequently found[4,7] to be about 0.55. n is more variable, though it is frequently[7] between 0.40 and 0.50. From data on an arc held stationary in an air stream,[5] n appears to be about 0.48 though lower values have been found for arcs moving in argon.[4] Using these values of m and n, it is possible to evaluate $c_\alpha B/E$ for current lower than those shown in Fig. 1 assuming that E does not vary with I. It is not possible to make allowance for this since for constant U_α, the V/I curve may have negative or positive slope depending upon the current, arc length, and velocity. For the copper rail data the current, arc length, and velocity,[8] at $M_\alpha = 0.20$, $c_\alpha B/E$ would become 0.074 for a 70-amp arc, which is the current for which data are quoted.[1] This value does in fact agree well with $k = 1.4$. The agreement between the source model and the particular experimental data quoted could, therefore, be somewhat fortuitous.

The result of Otis may be written (and in fact was initially derived with negligible induced voltage in the arc column) as

$$B/E = [1/c_p T_\alpha] U_\alpha \tag{2}$$

where c_p is the specific heat at constant pressure, T_α the freestream temperature. It may be deduced from Eqs. (61, 66, and 70) of Lord,[9] using a model that does not involve a source, that for an arc of low power gradient

$$B/E = [2\eta_2/(\hat{\phi} - \phi_\alpha)] U_\alpha \tag{3}$$

where η_2 is the mean coefficient of viscosity outside the arc and $(\hat{\phi} - \phi_\alpha)$ is a representative difference of heat flux potential between the arc and the freestream. By writing $\eta_2 = Prk_2/C_p$ and $(\hat{\phi} - \phi_\alpha) = k_2(\hat{T} - T_\alpha)$, where Pr is the Prandtl number, k_2 the mean thermal conductivity outside the arc and $(\hat{T} - T_\alpha)$ is a representative temperature difference, Eq. (3) becomes

$$B/E = [2Pr/C_p(\hat{T} - T_\alpha)] U_\alpha \tag{4}$$

This is directly comparable with equation (2). The presence of the Prandtl number in Eq. (4) indicates that this result is derived from a theoretical model totally different from that of Otis. Numerically, however, the biggest difference lies in the presence of the temperature difference $(\hat{T} - T_\alpha)$ which is probably an order of magnitude bigger than T_α (several thousand compared with several hundred degrees K). Hence, it might be expected that, for very low Mach numbers, a formula of the general form "B/E proportional to U" would be appropriate. It would be unwise however, to try to deduce the constant of proportionality from existing experimental measurements or expect the constant (when known) to apply for arcs at higher Mach numbers.

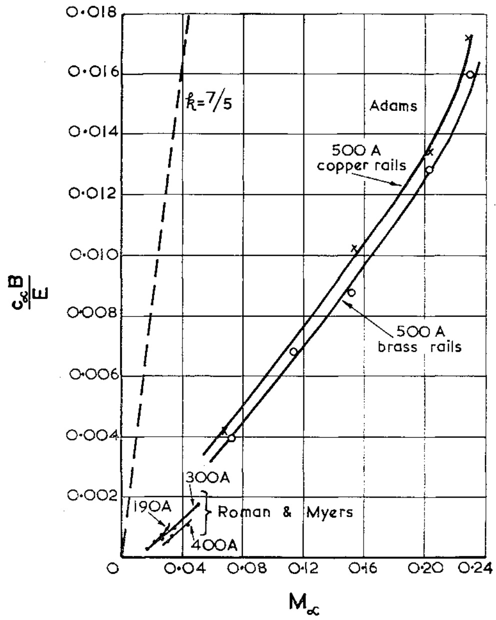

Fig. 1 Results for an arc held stationary in an air stream by a transverse magnetic field.

To summarize, it is suggested here that Otis' result is of limited application and that its theoretical basis is open to criticism, but it is pointed out at the same time that the result resembles one deduced by Lord for low Mach numbers. It should be recalled that both theories assume a column of infinite length, and the effects of arc length are of great importance in practice.[10]

References

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